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Partial barrier coverage: Using game theory to optimize probability of undetected intrusion in polygonal environments

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Abstract— **In this paper, we formalize the problem of partial barrier coverage, that is, the problem of using robot sensors (*guards*) to minimize the probability of undetected intrusion in a particular region by an *intruder*. We use ideas from noncooperative game theory together with previous results from complete barrier coverage – the problem of completely preventing undetected intrusion – to develop new methods that solve this problem for the specific case of bounded-range line-of-sight sensors in a two-dimensional polygonally-bounded region. Our solution constructs equilibrium strategies for the intruder and guards, and calculates the level of partial coverage.**

I. Introduction

In this paper, we address the problem of *partial barrier coverage*, the problem of minimizing the probability of a mobile entity (*intruder*) entering a specific area without being seen by a robot sensor (*guard*). Barrier coverage has applications in military security, building security [1], [2], robot herding [3], [4], and sensor networks [5], [6]. Specifically, we are looking at the problem of protecting a region in the plane using variable bounded-range line-of-sight detectors (*segment guards*). A segment guard can detect an intruder crossing its line of sight, but only within a certain range. This range is not a fixed parameter, but is set by the deployer. The deployer has a fixed bound on the total of all assigned ranges, which can be distributed to any number of segment guards. For the problem of partial barrier coverage, this bound is lower than the minimum range necessary to completely prevent undetected intrusion.

When deciding where to place guards, a deployer should consider the paths the intruder is likely to take. Similarly, when selecting an intrusion path, an intruder should consider the likely guard deployments. Because each entity considers the strategies of the other, and the two have opposite goals, the intruder/guard situation can be viewed as a noncooperative game. The outcome of this game is one if the intruder is detected, and zero if it reaches its goal undetected. Both opponents ultimately select probabilistic strategies; the expected outcome of this game is the probability that the guards will catch the intruder. The use of probabilistic strategies in security also appears in [7].

In this paper, we combine our results in complete barrier coverage for variable-length guards in polygonal environments [8] with noncooperative game theory results [9] to produce equilibrium strategies for both intruder and guards. The equilibrium strategies dictate the optimal probability distribution over guard deployments, and the game's value is the partial coverage value. This work allows us to (1) expand partial barrier coverage into a wider variety of workspaces, and (2) consider better intruder strategies when determining guard deployments.

This paper is laid out as follows. In Section II we review previous work in barrier coverage. In Section III we give definitions of terms and notations for barrier coverage, and in Section IV we give notation and basic concepts in game theory. In Section V we combine these to determine optimal strategies for intruder and guards in polygonal environments. In Section VI we describe future work, and in Section VII we give concluding remarks.

II. Previous Work

Barrier coverage is one of the three types of coverage [10]. The other two are *blanket coverage* and *sweep coverage*. The goal of blanket coverage [11]–[13] is to maximize the total area the robots can see. The goal of sweep coverage [14]–[16] is to maximize the set of points in a region that robots see as they move across it. Barrier coverage differs from the other two types in that its goal is minimizing unseen paths, rather than unseen points.

In the field of sensor networks, barrier coverage appears as moat construction, or intruder detection. Barriers across rectangular or annular regions have been generated using potential fields [17], incremental deployments [18], and sensor grids [5]. In general, random intruder motion is assumed while determining coverage values. Some approaches instead seek to determine the worst-case coverage levels. These approaches are related to the problem of *minimum exposure path* [19], [20], the problem of finding the path which is least likely to be seen by a sensor.

A related yet distinct problem is *complete barrier coverage* [8]. This is the problem of deploying guards to completely prevent the intruder from reaching his goal. This is similar to the previous related problems, with the added goal of ensuring sensors cover every path with probability 1. It is also related to separation problems from computational geometry. There are several problems of separating polygons into separate regions, either with line segments [21], circles [22], wedges [23], strips [23], or chords [24].

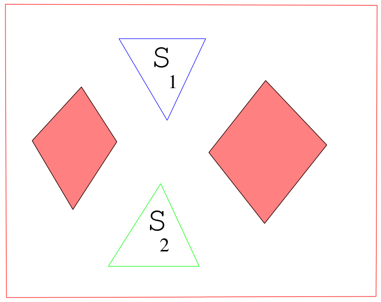

Fig. 1. Sample Barrier Problem Domain. The shaded regions are obstacles

While complete barrier coverage deals with the same issues of guards and intruder paths as partial barrier coverage, partial barrier coverage must also consider intruder strategies. A complete barrier covers all intrusion paths. A partial barrier covers an infinite set of paths, and misses an infinite set of paths. Determining what portion of paths are covered requires knowing the likely distribution of intrusion paths, i.e. the intruder's strategy.

III. Definitions

In this paper we focus on a point intruder moving in the plane. The intruder can only be in the obstacle-free workspace $\mathcal{W} \subset \mathbb{R}^2$, which is compact, connected, and bounded by polygons. The intruder is known to originate somewhere in the *start set* $\mathcal{S}_1 \subset \mathcal{W}$, and is trying to travel to some point in the *stop set* $\mathcal{S}_2 \subset \mathcal{W}$. Both $\mathcal{S}_1$ and $\mathcal{S}_2$ are compact and bounded by polygons. Figure 1 shows an example problem domain.

We define guards in terms of the regions they can see. We define each guard q_j to be a line segment whose interior lies entirely inside $\mathcal{W}$, and define r_j to be the length of this segment. This represents a guard that can see in one direction, up to a distance r_j, and cannot see through walls. We call these *segment guards*.

Consider a set of guards $q = \{q_1, \ldots, q_n\}$. We call this a *deployment*. A path from $\mathcal{S}_1$ to $\mathcal{S}_2$ (an *intrusion path*) that crosses any q_j is called a *detected intrusion*. An intrusion path that does not cross any guard in q is called an *undetected intrusion*. These terms indicate whether the intruder is caught by the guards while attempting to intrude. A deployment that guarantees there are no undetected intrusions is a *complete barrier*. Equivalently, a deployment is a complete barrier iff it separates $\mathcal{S}_1$ and $\mathcal{S}_2$ into separate connected components of $\mathcal{W}$.

In complete barrier coverage, we seek to find the complete barrier with minimum $\sum_{j=1}^{n} r_j$. We call the length of this minimum barrier W. In partial barrier coverage, we consider the situation where there is a limit on guard resources, i.e. $\sum r_j$ cannot exceed a set value R, where $R < W$. This restriction makes it impossible for the guards to completely guard $\mathcal{S}_2$ from intrusion from $\mathcal{S}_1$. We call a deployment in such a scenario a *partial barrier*.

IV. Game Theory

In this section, we describe how we model partial barriers using noncooperative game theory. We use notation and ideas from [9].

We describe partial coverage as a two-player static zero-sum game between the intruder $\mathbf{P1}$ and the deployer of the guards $\mathbf{P2}$. For $\mathbf{P1}$ an *action* is a path from $\mathcal{S}_1$ to $\mathcal{S}_2$. For $\mathbf{P2}$, an action is a guard deployment. The *outcome* of this game is 0 if the intruder intrudes undetected, and 1 if the guards detect him. $\mathbf{P2}$ wants to maximize the outcome; $\mathbf{P1}$ wants to minimize it.

To analyze these games, we determine the optimal strategies for both players. We start with *pure strategies*, wherein each player selects an action, and always plays that action. Let $A(\gamma, q)$ be the outcome of the game where $\mathbf{P1}$ selects action (intrusion path) γ and $\mathbf{P2}$ selects action (deployment) q. If $\mathbf{P1}$ wants to minimize this outcome in the worst case, he will select a γ^* that minimizes $\max_q A(\gamma^*, q)$. This guarantees an outcome of at most

$$\overline{V}(A) = \min_{\gamma} \max_{q} A(\gamma, q). \tag{1}$$

$\overline{V}$ is called the *upper value* of A. Similarly, if $\mathbf{P2}$ wants to maximize this outcome in the worst case, he will select a q^* that maximizes $\min_{\gamma} A(\gamma, q^*)$. This guarantees an outcome of at least

$$\underline{V}(A) = \max_{q} \min_{\gamma} A(\gamma, q). \tag{2}$$

$\underline{V}$ is called the *lower value* of A. If both players play these strategies, the game's outcome will always be between $\underline{V}$ and $\overline{V}$. If $\overline{V} = \underline{V}$, then γ^* and q^* form *equilibrium strategies* for A. The pair (γ^*, q^*) is also called a *saddle point* for *pure strategies*. The value of such a game is $V = \overline{V} = \underline{V}$. Each player can guarantee an outcome no worse than V by playing his equilibrium strategy, but cannot guarantee anything better. If a player plays a strategy that is not an equilibrium, then it is possible for the outcome to be worse for him.

Equivalently, (γ^*, q^*) is a saddle point iff for all γ and q,

$$A(\gamma^*, q) \leq A(\gamma^*, q^*) \leq A(\gamma, q^*). \tag{3}$$

In other words, a strategy is equilibrium iff one player deviating from the strategy (and not the other) cannot improve the outcome for him.

For every partial coverage game, every deployment has an intrusion path that avoids it. Therefore, $\underline{V} = 0$. Similarly, every intrusion path has a deployment that detects it. Therefore, $\overline{V} = 1$. Since $\underline{V} < \overline{V}$, there are no saddle points for pure strategies. To find equilibrium strategies, it is necessary to use *mixed strategies*, where each strategy is a probability distribution over a player's possible actions. Here γ and q are probability distributions over all possible actions. $A(\gamma, q)$ is now the expected outcome if $\mathbf{P1}$ plays probabilistic strategy γ, and $\mathbf{P2}$ plays probabilistic strategy q. $\overline{V}$, $\underline{V}$, and V are similarly defined with respect to *expected* outcome.

We will refer to the value of a partial coverage game as the *coverage value*. For mixed strategies this is the probability

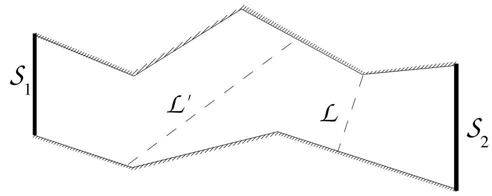

Fig. 2. Sample single corridor. $\mathcal{L}'$ is an arbitrary barrier, and $\mathcal{L}$ is the minimum barrier.

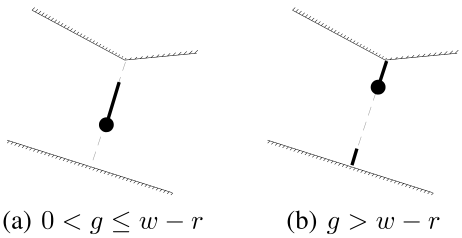

Fig. 3. Using $g \in [0, w]$ to place guards uniformly across a corridor.

of the guards catching the intruder, if both players use equilibrium strategies.

In the next section we apply game theory to increasingly complex environments. We determine equilibrium strategies for a single corridor, and then combine them with complete barrier results to construct equilibrium strategies for general polygonal environments.

V. Equilibrium strategies for guard/intruder games

A. One corridor

Consider a polygonal corridor with $\mathcal{S}_1$ at one end and $\mathcal{S}_2$ at the other end. This is a workspace in which there is a single homotopy class of paths from $\mathcal{S}_1$ to $\mathcal{S}_2$. $\mathbf{P2}$ has a guard length of r, to be apportioned out across one or more guards in the single corridor. Since this is partial coverage, we know $r < w$, where w is the length of the minimum barrier $\mathcal{L}$ (found using [8]). See Figure 2. $\mathbf{P1}$ selects a path from $\mathcal{S}_1$ to $\mathcal{S}_2$, and $\mathbf{P2}$ selects a guard deployment.

Here are the equilibrium strategies for this game:

- For $\mathbf{P1}$: select a point in $\mathcal{L}$ with uniform distribution, and select an intrusion path through this point.
- For $\mathbf{P2}$: any strategy such that every point in $\mathcal{L}$ is covered with probability $\frac{r}{w}$.

Here is an example of an equilibrium strategy for $\mathbf{P2}$. Since $\mathcal{L}$ has length w, it has a bijection with $[0, w]$. Select $g \in [0, w]$ with a uniform distribution, and use it to place the guard(s). If $g \in [0, w - r]$ is selected, place one guard across $[g, g + r]$. See Figure 3(a). If $g \in (w - r, r]$, place two guards: one across $[g, w]$, and one across $[0, r - w + g]$. See Figure 3(b).

Theorem 1: The given strategies for the one-corridor game are equilibrium strategies, with $V = \frac{r}{w}$.

Proof: Since any point in $\mathcal{L}$ has a $\frac{r}{w}$ probability of being covered, and each point has an equal probability of being traversed, the resulting expected outcome is $\frac{r}{w}$.

To show these are equilibrium strategies, we show that for each player, any other strategy produces an inferior worst-case expected outcome. For $\mathbf{P1}$, if the distribution is not uniform, there is a region of total length r with a probability higher than $\frac{r}{w}$ of selection. If $\mathbf{P2}$ places guards exclusively in this region, the outcome will be greater than $\frac{r}{w}$. Similarly for $\mathbf{P2}$, if the points in $\mathcal{L}$ do not all have the same probability of being guarded, there is a point with a probability lower than $\frac{r}{w}$ of coverage. If $\mathbf{P1}$ selects this point exclusively to traverse, the outcome will be less than $\frac{r}{w}$.

Now suppose the guards are placed across a different barrier $\mathcal{L}' \neq \mathcal{L}$. Since $\mathcal{L}'$ is not the minimum barrier, it has width $w' > w$. $\mathbf{P1}$ has no choice on this barrier; his best strategy is to cross it with a uniform distribution. This produces an expected outcome of $\frac{r}{w'} < \frac{r}{w}$. Therefore such a strategy is not equilibrium. Therefore all strategies that are not the pair given above are not equilibrium, so the given pair is a saddle point. ∎

B. General polygonal environments: Complete barrier coverage

We now consider general polygonal environments, which we can view like multiple intersecting corridors. To address this problem, we make use of our results in complete barrier coverage. In this section, we summarize our work in [8], which we will later use to determine partial barrier strategies.

1) Barrier Candidate Graph: We define a *barrier candidate graph* that contains all segments relevant towards finding the minimum complete barrier. These are the *barrier candidates*. This graph is related to the reduced visibility graph [25].

While there are an infinite number of line segments inside $\mathcal{W}$, we only consider a finite subset to be considered barrier candidates. Barrier candidates must have endpoints on the boundary of $\mathcal{W} - \mathcal{S}_1 - \mathcal{S}_2$, with interiors entirely inside $\mathcal{W} - \mathcal{S}_1 - \mathcal{S}_2$. A barrier candidate with an endpoint at an obstacle must be the shortest segment from the other endpoint to that obstacle edge. Barrier candidates with an endpoint at $\mathcal{S}_i$ $(i = 1, 2)$ must be tangent to that $\mathcal{S}_i$ component. If there are two barrier candidates which intersect the same homotopy classes of intrusion paths, the longer candidate is *redundant*, and removed from the graph. We also remove supporting tangents between $\mathcal{S}_1$ and $\mathcal{S}_2$, as well as separating tangents between components of $\mathcal{S}_1$ or between components of $\mathcal{S}_2$. Figure 4 shows the barrier candidate graph for the example domain in Figure 1.

In [8] we prove that in a polygonal workspace, the minimum variable-length segment barrier separating $\mathcal{S}_1$ from $\mathcal{S}_2$ consists only of segments from the barrier candidate graph. The proof exploits the fact that barrier candidates are local minima with respect to barrier components. This means that any barrier component that does not consist of barrier candidates can be replaced by a shorter barrier component. Therefore, determining the minimum complete barrier is a matter of searching the set of barrier candidates. To do this, we construct the connectivity network.

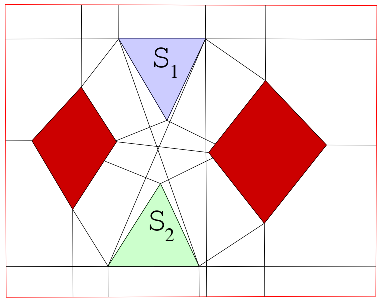

Fig. 4. Barrier candidate graph. The obstacles are shaded and unlabeled.

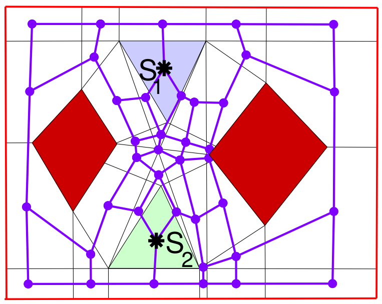

Fig. 5. Connectivity network overlaying barrier candidate graph from Fig. 4

2) Connectivity Network: The barrier candidate graph decomposes $\mathcal{W}$ into polygonal subregions. We construct a dual graph with a vertex for each region. Two vertices are connected with an edge iff their corresponding regions are adjacent. This edge is given a capacity equal to the length of the piece of the barrier candidate that separates the two subregions.

To this graph we add vertices s and t, corresponding to $\mathcal{S}_1$ and $\mathcal{S}_2$ respectively. Connect s to all the vertices corresponding to regions that intersect $\mathcal{S}_1$, and connect t to all the vertices corresponding to regions that intersect $\mathcal{S}_2$. These connections all have infinite capacity.

This graph, which we call the *connectivity network*, resembles a dual of the barrier candidate graph. The connectivity network for Fig. 4 is shown in Fig. 5. The connectivity of this graph reflects the connectivity of the workspace. Separating $\mathcal{S}_1$ from $\mathcal{S}_2$ is equivalent to separating s from t in the connectivity network. Therefore the minimum edge-cut that separates s from t corresponds to the minimum complete barrier.

Therefore to find the minimum complete barrier it suffices to solve the network flows min-cut problem, which is equivalent to the network flows maximum-flow problem [26]. The maximum flow of a network assigns a direction and flow value for each edge, such that (1) the flow value is less than or equal to the capacity; (2) for each vertex except for s and t, the flow in equals the flow out; and (3) the amount of flow out from s is maximized. The minimum cut of a network consists of all the edges where the flow is at capacity. This maximum flow can be found efficiently using augmenting paths [27] or preflows [28]. Figure 6 shows the minimum barrier of the sample domain. The dashed lines show the minimum cut for the connectivity network.

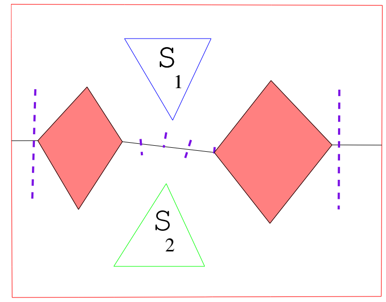

Fig. 6. Minimum Barrier. Dashed lines show corresponding edges in the dual graph

C. General polygonal environments: Partial barrier strategies

We use barrier candidates and the connectivity network to construct strategies for partial coverage in polygonal environments. Since any guard deployment that is not across barrier candidates only can be replaced by a better one that is across barrier candidates (Section V-B.1 plus Theorem 1), we will only consider guards across barrier candidates, and we will look at intruder paths based on which barrier candidates they cross.

This game can be viewed as a one-stage game, where $\mathbf{P1}$ selects a path through $\mathcal{W}$, and $\mathbf{P2}$ selects a deployment. It can also be viewed as a two-stage game, making use of the barrier candidate graph. In stage one, $\mathbf{P1}$ selects a path in the connectivity network; his action can be encoded as the set of barrier candidates his path intersects. Similarly, $\mathbf{P2}$ chooses a guard length r_i for every barrier candidate b_i (each b_i is of length w_i). Recall from Section III that the r_i values must sum to a predetermined R, where $R < W$, the length of the minimum barrier. In stage 2, $\mathbf{P1}$ chooses which point in each barrier candidate to cross, and $\mathbf{P2}$ chooses exactly where within each candidate b_i to put the r_i guards selected in stage 1.

Stage 2 is already known: Theorem 1 shows that if $\mathbf{P1}$ chooses a path and $\mathbf{P2}$ chooses a deployment such that b_i is the only barrier candidate that the intruder crosses and has guards on it, the outcome is $\frac{r_i}{w_i}$. Therefore, if $\mathbf{P1}$ chooses a path that crosses barrier candidates $\{b_1, \ldots, b_K\}$, then the game has outcome $1 - \prod_{i=1}^{K} \left(1 - \frac{r_i}{w_i}\right)$. This expression follows from the fact that the guards only need to see the intruder once for $\mathbf{P2}$ to win. We call this value $A(I, r)$, where $I = \{b_1, \ldots, b_K\}$ is the intruder's path, and $r = (r_1, \ldots, r_N)$ is the guard deployment.

With the second stage already determined, we focus on the first stage of the game. We build strategies for each player using results from minimum complete barriers. **P2** uses the minimum complete barrier B^*, which has total length W. **P1** uses the maximum flow through the connectivity network. Let $f(b_i)$ be the flow value through b_i in the maximum flow. Since the barrier candidate lengths are used for capacities, $f(b_i) \leq w_i$, with equality when $b_i \in B^*$.

The equilibrium strategies are:

- Select a path so that for **P1**: For each $i = 1, \ldots, N$, b_i is traversed with probability $\frac{f(b_i)}{W}$. Path I will be selected with the flow through I divided by W as its probability.
- For **P2**: For each $i = 1 \ldots, N$, set $r_i = \frac{w_i}{W} R$ iff $b_i \in B^*$, and $r_i = 0$ otherwise.

The flow through edges in B^* all flow from the components containing $\mathcal{S}_1$ to the components containing $\mathcal{S}_2$. Therefore, every **P1** path of nonzero probability – being a path with positive flow – traverses the minimum barrier exactly once. If it crosses edge $b_i \in B^*$, then the expected outcome is $\frac{r_i}{w_i} = \frac{R}{W}$. This is true regardless of which edge is selected.

We now show that these are equilibrium strategies.

Theorem 2: The strategies given above are equilibrium strategies for a polygonal environment, with $V = \frac{R}{W}$.

Proof: We show that the given strategies satisfy (3) by showing that if one player uses a different strategy and the other uses the given equilibrium strategy, the player choosing the different strategy is no better off.

First, consider alternative strategies for **P1**. Any path from $\mathcal{S}_1$ to $\mathcal{S}_2$ must cross B^* at least once. If **P2** uses the given strategy, then for any $b_i \in B^*$, $r_i = \frac{w_i}{W} R$. Therefore the resulting outcome will be at least $\frac{r_i}{w_i} = \frac{R}{W}$. This value will be higher if the path crosses B^* multiple times. Therefore, for **P1**, using other strategies produces an equal or higher outcome.

Now consider alternative strategies for **P2**. From Theorem 1 we already know that a deployment that is not across barrier candidates produces a smaller outcome. Therefore we write the deployment as $r = (r_1, \ldots, r_N)$, as above. **P2** places guards of total length r_i at barrier candidate b_i.

Given an intruder path I, the probability of being caught by a guard is

$$p_G(I) = 1 - \prod_{\{i|b_i \in I\}} \left(1 - \frac{r_i}{w_i}\right).$$

But this value must be no greater than the sum of the individual barrier candidates' detection probabilities, i.e.

$$1 - \prod_{i=1}^{K} \left(1 - \frac{r_i}{w_i}\right) \leq \sum_{i=1}^{K} \frac{r_i}{w_i},$$

Therefore we establish upper bounds on the expected outcome. Let $p(I)$ be the probability of **P1** selecting I. This is the flow through I divided by W. The expected outcome

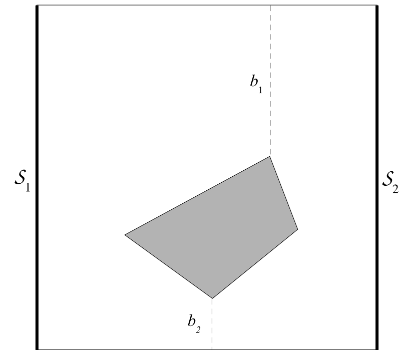

Fig. 7. Simple partial coverage example.

is therefore

$$\begin{aligned}
\sum_I p_G(I) p(I) &= \sum_I p(I) \left[1 - \prod_{\{i|b_i \in I\}} \left(1 - \frac{r_i}{w_i}\right)\right] \\
&\leq \sum_I p(I) \sum_{\{i|b_i \in I\}} \frac{r_i}{w_i} \quad (4) \\
&= \sum_{i=1}^{N} \frac{r_i}{w_i} \sum_{\{I|b_i \in I\}} p(I) \quad (5) \\
&= \sum_{i=1}^{N} \frac{r_i}{w_i} \frac{f(b_i)}{W} \\
&\leq \sum_{i=1}^{N} \frac{r_i}{W} \\
&= \frac{R}{W}.
\end{aligned}$$

The transition from (4) to (5) is a reordering of sums. (4) sums over each barrier candidate for every path, while (5) sums over each path for every barrier candidate.

Therefore, the given strategies satisfy (3), so they are equilibrium strategies.

■

See Figure 7 for a simple illustrative example. In this example, the minimum barrier consists of two components, b_1 and b_2. In this example, b_1 is three times as long as b_2, i.e. $w_1 = 3w_2$. The optimal strategy for **P1** is to traverse b_1 with probability $\frac{3}{4}$, and b_2 with probability $\frac{1}{4}$. Each b_i segment should be crossed at a point which is selected with a uniform distribution. This is true regardless of R. Similarly, **P2**'s optimal strategy is to cover b_1 with guards of total length 3 times the guards used to cover b_2. In other words, set $r_1 = \frac{3}{4}R$, and $r_2 = \frac{1}{4}R$.

Notice that the entire minimum barrier is traversed (by the intruder) or covered (by the guards) with a uniform distribution. Therefore, the strategies are equivalent to those if the barrier components were merged into one segment, like in Figure 8. This is true for any workspace.

VI. Future Work

This paper shows partial barrier coverage in a specific problem domain. We would like to expand the definition to

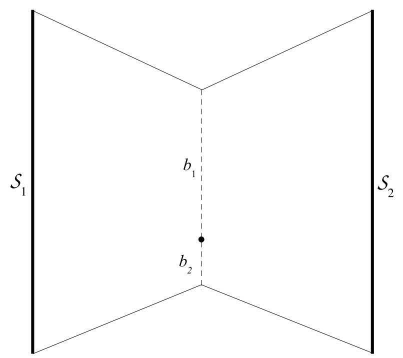

Fig. 8. Figure 7 with minimum barrier collapsed to one segment. The strategies are effectively the same.

other problem domains.

We would like to look at a greater variety of guards. This would include guards with fixed ranges, omnidirectional guards with and without range limits, guards with placement constraints, guards with noisy sensors, moving guards, and barriers composed of different types of guards. Similarly, we would like to look at intruders with shape and volume. We would also like to look at three-dimensional environments, with and without gravity.

We would also like to look at strategy constraints for both players. For the intruder this could involve motion constraints. For guards this could involve alternative deployment methods, like random deployments with and without guard motion. Each limitation on a player's strategy should create a change in his opponent's strategy.

VII. Conclusion

In this paper, we have shown how to combine methods in complete barrier coverage with with noncooperative game theory to produce equilibrium strategies for both intruder and guards in polygonal environments. We have shown that the equilibrium strategies for both players are to distribute their actions evenly across the minimum complete barrier, and that the resulting coverage value is the ratio of available guard resources to the minimum complete barrier length (i.e. $\frac{R}{W}$).

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